Chapter 34
A Practical Reduced-Rank Anti-Jamming Algorithm Based on Variable Diagonal Loading Method

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Abstract In the space–time anti-jamming practice of GNSS, the amplitude-phase error of array antenna and the relative motion of interference source might cause position deviation of null or theoretical null in anti-jamming algorithms, and consequently to affect the anti-jamming performance of anti-jamming algorithm severely. For this problem, a space–time 2D anti-jamming algorithm based on reduced-rank variable diagonal load was proposed. This algorithm is purposed to widen and correct the established null under the orthogonality principle of signal subspace and noise subspace, thereby to eliminate the effect to anti-jamming algorithm resulting from the space domain mismatch of signal subspace and the mismatch of signal covariance matrix and eventually to improve the robustness of anti-jamming algorithm. The simulation result validated the correctness of the theoretical analysis and the algorithm robustness.

Keywords Diagonal load · Space–time adaptive · Steering vector · Power inversion

34.1 Section Heading

The electromagnetic environment of GNSS is becoming increasingly complicated. The receiving system of GNSS usually uses space–time 2D anti-jamming algorithm for adaptive signal processing. However, this algorithm might be affected

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J. Sun et al. (eds.), *China Satellite Navigation Conference (CSNC) 2013 Proceedings*, Lecture Notes in Electrical Engineering 243,
DOI: 10.1007/978-3-642-37398-5_34, © Springer-Verlag Berlin Heidelberg 2013

due to vibration or motion of antenna receiving platform, quick change of interference location or array-element position error resulting from poor antenna manufacturing quality or installation-sourced mutual coupling, consequently causing mismatch of null position and jamming direction [1] or even complete failure of the anti-jamming algorithm.

To implement the space–time anti-jamming technique of GNSS, it is required to eliminate the affect to space–time 2D anti-jamming algorithm resulting from amplitude-phase error and receiver or interference motion. The feasible solution is to widen the null around the jamming signal with more robust adaptive processing algorithm. A common technique widening the null of jamming signal is diagonally-loading method. Diagonal loading algorithm is also known as artificial noise injection method [2]. The basic idea of this algorithm is to establish wider null in the jamming direction so as to avoid the interference location moving out of the null in the weight processing period, thereby to suppress the interference effectively. The method has good robustness in the case of small snapshot number and signal response error of random array [3]. It was firstly proposed by B. D. Carlson in 1988 [2], and later, Ma and Goh proposed variable diagonal loading method [4]. But all these loading algorithms need matrix inversion, which require complex computation and is not in favor of real-time implementation. After Goldstein proposed reduced-rank multi-level Wiener filtering algorithm [5], some researches were given to diagonal loading anti-jamming method with multi-level Wiener filtering in the backward synthetic process [6].

Based on an in-depth analysis of diagonal loading algorithm and making full use of the orthogonality of interference subspace and noise subspace, this paper introduced the variable diagonal loading method into the simplified reduced-rank multi-level Wiener filtering space–time 2D anti-jamming algorithm. In addition to algorithm principle and derivation process, the effectiveness of this new algorithm was validated with simulation experiment.

34.2 Diagonal Loading Anti-Jamming Principle

As its name suggests, diagonal loading method is defined as the algorithm that introduce a certain noise constant along the diagonal line of the covariance matrix of received signal and then substitute the regular sampling covariance matrix with diagonal loading covariance matrix. This method remains good robustness in the condition of small snapshot number.

$$R_{x,M} = \frac{1}{K}\sum_{k=1}^{K} X(k)X^{H}(k) + \sigma_L^2 I \tag{34.1}$$

where, $R_{x,M}$ represents the covariance matrix after diagonal loading; $X(k)$ represents the received satellite navigation signal; M represents the element number of receiving array; K represents the length of the received data given with covariance computation; I represents unit matrix; σ_L^2 represents diagonal loading quantity.

In space–time 2D anti-jamming applications, the algorithm based on linear-constraint minimum variance (LCMV) criterion [7] is very popular. Taking advantage of the characteristic that navigation signal power is far below noise and interfering power, it constrains the space domain and time domain simultaneously under the condition of space–time joint processing and adjust the weight to have the minimum output signal variance, so as to weaken the jamming energy significantly. The essence of diagonal loading space–time 2D anti-jamming algorithm is to add one secondary compensation term to adjust the weight vector for the objective function of linear constraint minimum variance algorithm [7], which is expressed as follows:

$$\begin{cases} \underset{w}{\text{Min}} & w^{\text{H}}Rw + \sigma_L^2 w^{\text{H}}w \\ \text{s.t.} & S^{\text{H}}w = b \end{cases} \tag{34.2}$$

where, σ_L^2 is the loading value.

Based on the Lagrange in equation, the space–time 2D anti-jamming optimal weight under this criterion is expressed as follows:

$$W_{opt} = \left(S^H(R + \sigma_L^2 I)^{-1}S\right)^{-1}(R + \sigma_L^2 I)^{-1}S. \tag{34.3}$$

where, W_{opt} W_{opt} represents $MP \times 1$th dimension vector; M represents the element number of array antenna; P represents the number of time cells; R represents the covariance matrix of $MP \times MP$-dimension received signal; $\boldsymbol{S}$ represents the constrained vector of $MP \times 1$ dimension. When the signal direction is unknown, $\boldsymbol{S} = [1, 0, \cdots, 0]^T$. The output signal, after anti-jamming processing, can be expressed as follows:

$$\boldsymbol{y} = \boldsymbol{W}^H\boldsymbol{X} \tag{34.4}$$

Diagonal loading algorithm is able to enhance the performance of space–time 2D anti-jamming algorithm when random array signal response error exists. By diagonal loading and choosing appropriate diagonal loading quantity, larger characteristic value corresponding to jamming signal will not be affected significantly, but the small characteristic value corresponding to noise signal will have an additional noise substrate σ_L^2 σ_L^2. The diagonally-loaded noise characteristic values will be distributed around σ_L^2 σ_L^2, which can reduce the diffusion level of noise characteristic values and thereby to reduce the affect of noise characteristic vector to adaptive weight vector [6].

34.3 Implementation Process of Diagonal Loading Reduced-Rank Filtering Algorithm

The multi-level Wiener filter uses a nested link composed of scalar Wiener filters that has very good reduced-rank processing capacity and does not require resolution to the characteristic values of covariance matrix. This feature gives it lower complexity of computation; therefore, it is an important breakthrough in reduced-rank adaptive filtering technology.

Multi-level Wiener filtering technique takes the system as dual input and implement it with multi-level resolution on the basis of regular cross-correlation. To lower the noise diffusance and the affect of loading quantity to large characteristic values and w_d^D, a processing method similar to diagonal loading method was considered for covariance matrix R_{X_0}, i.e., to add one more diagonal matrix to the covariance matrix R_D after reduced-rank processing and replace the covariance matrix R_D before diagonal loading with the diagonally loaded one.

Since the optimum block matrix $B_i = I - t_i t_i^H$, $[t_1, t_2, \ldots, t_D]$ belong to orthonormal vectors. Therefore, it is allowed to complete three diagonalization to the diagonally-loaded covariance matrix R_{dl} with matrix T_D.

$$R_D = T_D^{\mathrm{H}} R_{dl} T_D = T_D^{\mathrm{H}} (R_{X_0} + \gamma I) T_D = \begin{bmatrix} r_{1,1}+\gamma & r_{1,2} & & \\ r_{1,2}^* & r_{2,2}+\gamma & \ddots & \\ & \ddots & \ddots & r_{D-1} \\ & & r_{D-1,D}^* & r_{D,D}+\gamma \end{bmatrix} \tag{34.5}$$

I.e., the diagonal loading effect to rank-reduced covariance matrix R_D is equivalent to that to the covariance matrix R_{X_0} before reduced-rank processing is given.

Since R_D R_D is a Hermitian matrix, the rank-reduced matrix R_D is a given Hermitian matrix and the column vector of rank-reduced matrix T_D forms Krylov subspace. Therefore, it is allowed to derive the iteration structure of weighted multi-level Wiener filtering with Lanczos iterative algorithm [8].

$$R_D = \begin{bmatrix} T_{D-1}^{\mathrm{H}} R_{dl} T_{D-1} & & \mathbf{0} \\ & & r_{D-1} \\ \mathbf{0}^{\mathbf{T}} & r_{D-1,D}^* & r_{D,D}+\gamma \end{bmatrix} \tag{34.6}$$

The cross correlation with the desired signal $d_0(k)$ is $\mathbf{r}_{\mathbf{X}_T d_0}^{(D)} = \mathbf{T}_D^{\mathrm{H}} \mathbf{r}_{\mathbf{X}_0 d_0} = \begin{bmatrix} \|\mathbf{r}_{\mathbf{X}_0 d_0}\| \\ \mathbf{0} \end{bmatrix} \in \mathbb{R}^D$, where $\mathbb{R}^D$ represents real D-dimension space. The covariance matrix $R_{D-1} = T_{D-1}^{\mathrm{H}} R_{\mathbf{X}_0} T_{D-1}$ is already known, then the new term in R_D is $r_{D-1,D} = t_{D-1}^{\mathrm{H}} R_D t_D$. To solve Wiener filter $W_D = R_D^{-1} r_{\mathbf{X}_T d_0}^{(D)} \in \mathbb{C}^D$ with $r_{D,D} = t_D^{\mathrm{H}} R_D t_D$, considering only the first element in $r_{\mathbf{X}_T d_0}^{(D)}$ is not zero, only the first column in $C^{(D)} = R_D^{-1} = \left[c_1^{(D)}, c_2^{(D)}, \ldots, c_D^{(D)}\right]$ is therefore prerequisite. With reference to the matrix inversion lemma [9] of partitioned matrix, it is allowed to use recursion method to work out $C^{(D)}$.

$$C^{(D)} = \begin{bmatrix} C^{(D-1)} & 0 \\ 0^{\mathrm{T}} & 0 \end{bmatrix} + \beta_D^{-1} b^{(D)} b^{(D),\mathrm{H}} \tag{34.7}$$

where, $b^{(D)} = \begin{bmatrix} -r_{D-1,D} c_{D-1}^{(D-1)} \\ 1 \end{bmatrix} \in \mathbb{C}^D$, $\beta_D = r_{D,D} - \left|r_{D-1,D}\right|^2 c_{D-1,D-1}^{(D-1)}$ and $c_{D-1,D-1}^{(D-1)}$ is the last element in the last column $c_{D-1}^{(D-1)}$ of $C^{(D-1)}$. Therefore, the new first column $c_1^{(D)} \in \mathbb{C}^D$ at the Dth step may be expressed as:

$$c_1^{(D)} = \begin{bmatrix} c_1^{(D-1)} \\ 0 \end{bmatrix} + \beta_D^{-1} c_{1,D-1}^{(D-1),*} \begin{bmatrix} \left|r_{D-1,D}\right|^2 c_{D-1}^{(D-1)} \\ -r_{D-1,D}^* \end{bmatrix} \tag{34.8}$$

where, $c_{1,D-1}^{(D-1)}$ is the first element in the last column $c_{D-1}^{(D-1)}$ of $C^{(D-1)}$. The updating of Wiener filter W_D at the Dth step requires the first column $c_1^{(D-1)}$ at the $D-1$ step and the two new terms of covariance matrix, i.e., $r_{D-1,D}$ and $r_{D,D}$. The updating of the last matrix column only depends on the last column of the previous matrix and the new term of matrix R_D.

$$c_D^{(D)} = \beta_D^{-1} \begin{bmatrix} -r_{D-1,D} c_{D-1}^{(D-1)} \\ 1 \end{bmatrix} \tag{34.9}$$

Therefore, it is required to update vectors $c_1^{(D)}$ and $c_D^{(D)}$ at every step in a complete interactive process. The MSE of multi-level Wiener filter is defined as follows, where D represents the level.

$$MSE^{(D)} = \mathrm{E}\left[\left|e_0(k)\right|^2\right] = \mathrm{E}\left[\left|d_0(k) - W_{MWF}^{(D)} X_0(k)\right|^2\right] \tag{34.10}$$

It is easy to know that $MSE^{(D)}$ may be represented by the first element $c_{1,1}^{(D)}$ of $c_1^{(D)}$.

$$MSE^{(D)} = \sigma_{d_0}^2 - \|r_{\mathbf{X}_0 d_0}\|_2^2 c_{1,1}^{(D)} \tag{34.11}$$

$$\omega_0^{(D)} = \left\|r_{x_0,d_0}\right\|_2 T^{(D)} c_1^D \tag{34.12}$$

As shown in the above formula derivation process, only $\beta_D = r_{D,D} - \left|r_{D-1,D}\right|^2 c_{D-1,D-1}^{(D-1)}$ is directly affected by diagonal loading process in the above procedure. Since diagonal loading only affects the value of $r_{i,i}$ and no change happens to the value of $r_{i-1,i}$, the phase difference γ between the diagonally-loaded

β_i and the not-diagonally-loaded β_i is {FLD6} and γ represents diagonal loading quantity.

β_i will decrease along with the increase of the number of reduced-rank dimensions; the effect of diagonal loading quantity γ to β_i therefore become more and more stronger along with the number of reduced-rank dimensions. The diagonal loading quantity reduces the updating weight of high-order c_1^i and c_D^i, thereby to affect the contribution of the number of rank-reduced dimensions to the final weight. The more large the diagonal loading quantity is, the more noticeable such offsetting effect is. Before the number of rank-reduced dimensions reaches the optimum, its side effect will appear, i.e., lowering the output SINR. However, if the loading level is too low and the performance after and before diagonal loading gives no difference, the diagonal loading effect is not satisfactory.

Therefore, to achieve satisfactory diagonal loading effect, it is required to manage the diagonal loading quantity γ to decrease at every step of the algorithm along with the increase of the iteration times.

$$\gamma_i = \frac{\beta_i}{\beta_{i-1}} \gamma_{i-1} \tag{34.13}$$

The typical level loading method often used in engineering is 5–10 dB higher than the noise background. The procedure to implement diagonal-loading-based adaptive rank-reduced multi-level nesting Wiener filtering algorithm is specific as follows:

1. Data initialization:

Choose the maximum number of dimensions D.

Set $t_0 = 0$, $d_0(k) = S^H X_{MN}(k)$, $X_0(k) = X_{MN}(k) - Sd_0(k)$, $r_{1,1} = E\{d_1(k) d_1^*(k)\}$

$$c_{first}^{(1)} = r_{1,1}^{-1} \quad c_{last}^{(1)} = r_{1,1}^{-1}$$

$$\gamma_0 = \sigma_L^2$$

2. Interactive process:

$$i = 2, \ldots, D$$

$$r_{X_i d_i} = \frac{1}{K} \sum_{k=1}^{K} X_i(k) d_i^*(k)$$

$$t_{i+1} = r_{X_i d_i} \Big/ \sqrt{r_{X_i d_i}^* r_{X_i d_i}}$$

$$r_{i,i} = \frac{1}{K} \sum_{k=1}^{K} |d_i(k)|^2$$

$$r_{i-1,i} = \frac{1}{K}\sum_{k=1}^{K} d_{i-1}(k)d_i^*(k)$$

$$\beta_i = r_{i,i} - \left|r_{i-1,i}\right|^2 c_{last,i-1}^{(i-1)}$$

$$\gamma_i = \frac{\beta_i}{\beta_{i-1}}\gamma_{i-1}$$

$$\widehat{\beta}_i = \beta_i + \gamma_i$$

$$c_{first}^i = \begin{bmatrix} c_{first}^{(i-1)} \\ 0 \end{bmatrix} + \widehat{\beta}_i^{(i-1)} c_{first,1}^{(i-1)} \cdot \begin{bmatrix} \left|r_{i-1,i}\right|^2 c_{first}^{(i-1)} \\ -r_{i-1,i}^* \end{bmatrix}$$

$$c_{last}^i = \widehat{\beta}_i^{(i-1)} \cdot \begin{bmatrix} \left|r_{i-1,i}\right|^2 c_{last}^{(i-1)} \\ 1 \end{bmatrix}$$

$$MSE^{(i)} = \sigma_{d_0}^2 - \left\|r_{x_0,d_0}\right\|_2^2 c_{first}^{(i)}$$

3. Weight estimation:

$$T^{(D)} = [t_1, \ldots, t_\Delta]$$

$$\omega_0^{(D)} = \left\|r_{x_0,d_0}\right\|_2 T^{(D)} c_{first}^{(i)}$$

$$W_{MWF} = \begin{bmatrix} 1 \\ -\omega_0^D \end{bmatrix}$$

34.4 Analysis of Performance Simulation

To validate the algorithm performance, the following simulation was given. In the simulation, a 5-element circular array (with the center of circle is provided) of GNSS receiver was set. One of the elements was at the centre of the circle and other four elements were uniformly distributed around the circumference of the circle. The radius of the circle was $\sqrt{2}\lambda/4$, and λ was the signal wavelength. In this case, the interval between the adjacent elements along the circumference was $\lambda/2$. $\theta \in [0, \pi/2]$ and $\varphi \in [0, 2\pi]$ were the elevation angle and the azimuthal angle of the received signal. Let's assume that P code is used as the spread-spectrum code for the modulation of satellite navigation signal, the noise is additive white Gaussian noise, the code rate is 10.23 MHz/s, the input SNR is -15 dB and all the

SINR is 40 dB. Under the condition of meeting the performance requirements, set the number of delay tap $N = 3$ considering the purpose of simplifying computation workload and improving the real-timeness. The elevation angle of the incoming signal is 80° and the azimuthal angle is 180°. Zero IF baseband signal was used directly in the simulation, which was represented with normalized frequency bandwidth in the chart. Normalized frequency bandwidth refers to the ratio of actual interference bandwidth to receiver bandwidth.

Simulation Experiment 1: output SINR changing along with the angular error.

Provide a single broadband jamming source and set the number of snapshots L = 500. In the real applications, the estimation error of the signal arrival direction might cause mismatch between the steering vector of the estimated desired signal and the steering vector of the real signal source; therefore, the pointing error of the desired signal was set to a variation range of −5° to 5°.

Figure 34.1 shows the curves of output SINR changing along with the pointing error in three algorithm cases: rank-reduced multi-level Wiener filtering algorithm without diagonal loading under linear constraint, rank-reduced algorithm based on fixed diagonal loading and the algorithm proposed in this paper. As shown in the figure, the algorithm proposed by this paper provided the best performance, the fixed diagonal loading rank-reduced algorithm was next, and both algorithms were superior to the rank-reduced multi-level Wiener filtering algorithm without diagonal loading. Conclusively, adaptive diagonal loading algorithm is able to effectively improve the filtering performance of anti-jamming algorithm in case of pointing error.

Simulation Experiment 2: Output SINR changing along with the number of rank-reduced dimensions.

Provide single broadband jamming source. Set the number of snapshots to 500. Simulate the output SINR changing along with the number of snapshots in three algorithm cases: rank-reduced multi-level Wiener filtering algorithm without

Fig. 34.1 Curve of output SINR changing along with pointing error

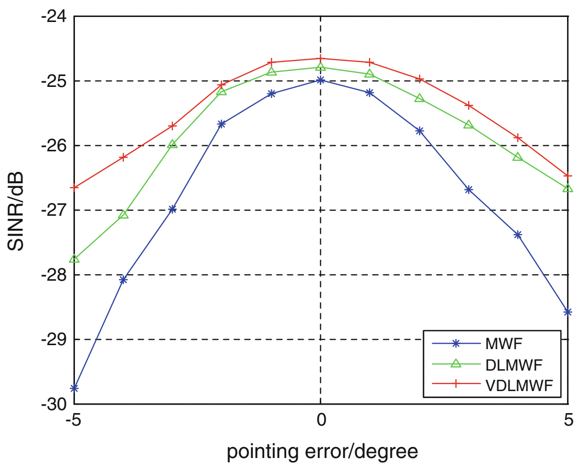
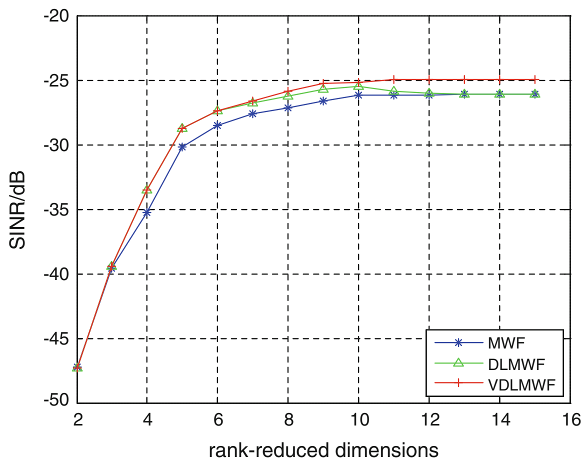


Fig. 34.2 Curve of output SINR changing along with rank-reduced dimensions



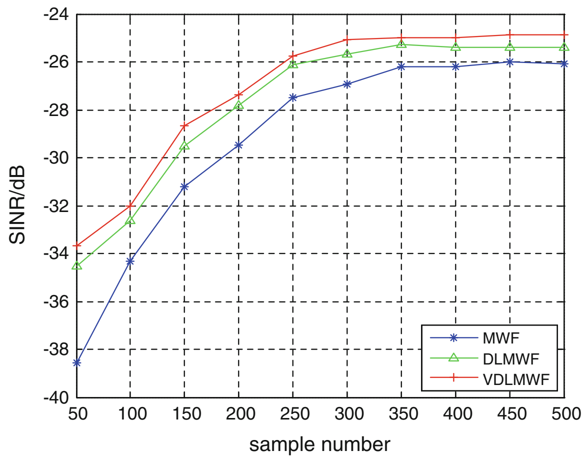
diagonal loading, the one based on fixed diagonal loading and the one based on variable diagonal loading (Fig. 34.2).

As shown in the figure, the algorithm based on variable diagonal loading provided the best performance, the algorithm based on fixed diagonal loading gave somewhat declined performance when the number of rank-reduced dimensions got to some extent, and both the algorithms were superior to the rank-reduced space–time anti-jamming algorithm without diagonal loading in filtering performance.

Simulation Experiment 3: Output SINR changing along with the number of snapshots.

Provide a single broadband jamming source. Set the number of snapshots to 500 from 50. Simulate the variation curves of output SINR changing along with the number of snapshots in the three algorithm cases (Fig. 34.3).

Fig. 34.3 Curve of output SINR changing along with sample number



As shown in the figure above, the algorithm proposed in this paper was superior to the anti-jamming method based on fixed diagonal loading algorithm and the one without diagonal loading. The algorithm proposed in this paper supports adaptive selection of loading quantity. It is able to reduce the diffusance of noise characteristic value without affecting the interference characteristic value. Therefore, this algorithm has better anti-jamming filtering performance. The computational complexity of the algorithm proposed in this paper is approximately $O\left((MN)^2 + 5(MN)\right)$, which is much reduced in comparison with the one using matrix inversion diagonal loading algorithm $O\left((MN)^4 - 2(MN)^3 + 2(MN)^2 - 1\right)$. Therefore, the former has better real-timeness and engineering reliability.

34.5 Conclusion

To reduce the effect of motion and error to the robustness of space–time adaptative anti-jamming algorithm and improve the performance of space–time 2D anti-jamming algorithm, this paper proposed a variable diagonal loading rank-reduced multi-level Wiener anti-jamming algorithm based on diagonal loading method. This algorithm does not require matrix inversion and thereby simplifies the computational complexity. Comparing with the algorithm based on fixed diagonal loading method, it has better anti-jamming performance and robustness. Under the condition of small snapshot number, it still show good anti-jamming performance. The simulation result also demonstrated the excellent performance of this algorithm. It is of important the theoretical value and engineering applications value for the design of GNSS anti-jam receiver.

Acknowledgment The work of this article is supported by National Science and Technology Ministry (No. 2011BAH24B04).

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